DISCHARGE OF A GAS-SATURATED LIQUID THROUGH A VENTURI

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Equations are derived for determining the flow rate when a gas-saturated liquid discharges through a venturi.

During an experimental evaluation of water–air injectors on an apparatus in which the water entering the operating nozzle is saturated with dissolved air, a discharge mode where the flow rate of water is independent of the pressure of the medium into which it is discharged can be observed. An explanation of this phenomenon is that air dissolved in the water escapes, as a result of which a two-phase compressible mixture is discharged from the nozzle. In order to derive an equation for the flow rate of the liquid which will also take into account the effect of the dissolved gas, we make the following assumptions: the discharge is an equilibrium process, i.e., the concentration of gas dissolved in the liquid obeys Henry's law at any section of the venturi, the Henry coefficient depending only on the temperature; the resulting gas–liquid mixture is homogeneous and isotropic.

The flow rate of the liquid during the subcritical discharge of a gas–liquid mixture will be found from the Bernoulli equation:

$$\frac{w^2}{2} + \int \frac{dp}{\rho_{mix}} = \text{const},$$

where the density can be expressed as

$$\rho_{mix} = \frac{\rho_L}{1 + \delta \frac{p' - p}{p}},$$

where $\delta = \alpha(1 + T_L/273)$. Then, according to [1], the Bernoulli equation becomes

$$\frac{w^2}{2} + (1 - \delta)\frac{p}{\rho_L} + \delta \frac{p'}{\rho_L} \ln p = \text{const}. \quad (1)$$

Writing for the flow rate of the liquids:

$$Q_L = \frac{fw}{1 + \delta \frac{p' - p}{p}}$$

and inserting the velocity from Eq. (1), we obtain

$$Q_L = \frac{f\sqrt{2(1-\delta)\frac{p_0 - p}{\rho_L} + 2\delta \frac{p'}{\rho_L} \ln \frac{p'}{p}}}{1 + \delta \frac{p' - p}{p}}. \quad (2)$$

The condition for critical discharge of the liquid is that c = w, where, according to [2],

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$$c=\sqrt{\frac{v_{\text{mix}}}{\frac{\varphi_L v_L}{c_L^2}+\frac{\varphi_G v_G}{c_G^2}}}. \tag{3}$$

Knowing that $\varphi_L = 1-\varphi_G$, $v_{mix} = v_L/(1-\varphi_G)$, $v_G = RT/p$, and $c_G^2 = kgRT$, we have

$$c=\sqrt{\frac{1}{\frac{(1-\varphi_G)^2}{c_L^2}+\frac{\varphi_G(1-\varphi_G)}{v_L kgp}}}. \tag{4}$$

With a negligibly small error, we may assume the liquid to be incompressible so that Eq. (4) becomes

$$c=\sqrt{\frac{kp}{\rho_L\varphi_G(1-\varphi_G)}}. \tag{5}$$

Into Eq. (5) we insert the value of φ_G in terms of the Bunsen absorption coefficient

$$\varphi_G=\frac{\delta\frac{p'-p}{p}}{1+\delta\frac{p'-p}{p}}. \tag{6}$$

Then

$$c=\sqrt{\frac{kp\left(1+\delta\frac{p'-p}{p}\right)^2}{\rho_L\delta\frac{p'-p}{p}}}. \tag{7}$$

With (1) and (7), the equality c = w becomes

$$2(1-\delta)\frac{p_0-p^*}{\rho_L}+2\delta\frac{p'}{\rho_L}\ln\frac{p'}{p^*}=\frac{kp^*\left(1+\delta\frac{p'-p^*}{p^*}\right)^2}{\rho_L\delta\frac{p'-p^*}{p^*}}. \tag{8}$$

After a few transformations, we have

$$\frac{1}{(p^*)^2}\left[\frac{2\delta(1-\delta)}{k}p_0p'+\left(\frac{2\delta^2}{k}\ln\frac{p'}{p^*}-\delta^2\right)(p')^2\right]-\frac{1}{p^*}\left\{\left[\frac{2\delta(1-\delta)}{k}+\frac{2\delta^2}{k}\ln\frac{p'}{p^*}+2\delta(1-\delta)\right]p'\right.$$
$$\left.+\frac{2\delta(1-\delta)}{k}p_0\right\}+\frac{2\delta(1-\delta)}{k}-(1-\delta)^2=0. \tag{9}$$

For the case where $p' = p_0$, Eq. (9) becomes

$$\left(\frac{p_0}{p^*}\right)^2-\frac{p_0}{p^*}\,\frac{4\delta(1-\delta)+2\delta^2\ln\frac{p_0}{p^*}+2\delta(1-\delta)k}{2\delta(1-\delta)+2\delta^2\ln\frac{p_0}{p^*}-k\delta^2}+\frac{2\delta(1-\delta)-(1-\delta)^2k}{2\delta(1-\delta)+2\delta^2\ln\frac{p_0}{p^*}-k\delta^2}=0. \tag{10}$$

In this way, from the solution to Eq. (9) or (10) we can find the critical pressure ratio for a venturi during the discharge of a gas-saturated liquid. The values of the critical pressure ratio $\beta = p^*/p_0$ shown in Table 1 were obtained by using Eq. (10) for water saturated with air and with carbon dioxide.

The flow rate of the liquid during critical discharge is

$$Q_L=\frac{fc}{1+\delta\frac{p'-p^*}{p^*}}=f\sqrt{\frac{kp^*}{\rho_L\delta\frac{p'-p^*}{p^*}}}. \tag{11}$$

The equations derived here agree with experimental data obtained using actual water–air injector nozzles built as venturis with a $\theta = 13°24'$ aperture angle and a conical converging section of length L = 0.25 m. It must be noted that these equations are valid for channels shaped so that the liquid remains in them long enough for the gas to be desorbed from the solution. The time that a liquid remains in the channel can be expressed, with sufficient accuracy, as a function of the channel geometry:

TABLE 1. Values of the Critical Pressure Ratio β, Calculated from Eq. (10)

Dissolved gas	T, °C				
	5	10	15	20	25
Air	0,142	0,138	0,131	0,123	0,115
Carbon dioxide	0,390	0,445	0,510	0,605	0,690

$$t=\int\frac{dL}{w}=\frac{(r_1^3-r^3)\left(1-\delta\frac{p'}{p^*}\right)}{r^2\operatorname{tg}\frac{\theta}{2}\sqrt{(1-\delta)\frac{p_0-p^*}{\rho_L}+\delta\frac{p'}{\rho_L}\ln\frac{p'}{\rho^*}}},$$

and it amounts to 0.05-0.08 sec for the venturis described here. In view of the insufficient knowledge we have about gas desorption from a solution, it is impossible now to establish the maximum length of time up to which the discharge equation will remain valid. Experiments have shown that a time t = 0.05-0.08 sec is sufficient to reduce the pressure of a saturated solution from p' to p* in the test nozzles.

NOTATION

p_0 is the pressure at the venturi outlet;
p is the pressure in the medium into which discharge occurs;
p' is the pressure of a saturated solution;
p* is the pressure at the critical venturi section;
ρ is the density;
α is the Bunsen absorption coefficient;
β is the critical pressure ratio;
ω is the velocity of liquid flow;
R is the gas constant;
T is the temperature;
c is the velocity of sound in the gas–liquid mixture;
f, r are the area and radius of the venturi outlet section;
r_1 is the radius of the venturi cross section where the pressure is p'.

Subscripts

G denotes gas;
L denotes liquid;
mix denotes gas–liquid mixture.

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